11th Workshop on Operator Theoretic Aspects of Ergodic Theory

Wuppertal, November 24–25, 2023.

Titles and Abstracts

Version November 13, 2023

Michael Baake: *Hats, CAPs and Spectres*

Abstract: The title says it all ...

Catalin Badea: *Around Furstenberg's times p, times q conjecture*

Abstract: For each integer $n \geq 1$, denote by T_n the map $x \mapsto nx \mod 1$ from the circle group $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ into itself. Let $p, q \geq 2$ be two multiplicatively independent integers. Using Baire Category arguments, we show that generically a T_p-invariant probability measure on $\mathbb{T}$ with no atom has some large Fourier coefficients along the sequence $(q^n)_{n\geq 0}$. This disproves a conjecture of Furstenberg and complements previous results of Johnson and Rudolph. In the spirit of previous work by Meiri and Lindenstrauss-Meiri-Peres, we study generalisations of our main result to certain classes of sequences $(c_n)_{n\geq 0}$ other than the sequence $(q^n)_{n\geq 0}$, and also investigate the multidimensional setting. This is joint work with Sophie Grivaux.

Karma Dajani: *The matching phenomenon*

Abstract: Matching is a mysterious phenomenon which has recently been observed for certain parametrized interval maps in the deterministic and random settings. For example, α-continued fractions, α-symmetric binary maps as well as α-golden mean maps. Matching is the property that for each discontinuity point the orbits of the left and right limit merge after some finite number of steps and that the (expected value of the) derivatives of both orbits are also equal at that time; this assures the stability of this phenomenon under small perturbations of the parameter. Since most of the dynamical behaviour of systems are encoded in the possible trajectories of the discontinuity points, knowledge on when and how matching occurs can help in finding explicit expression for the natural invariant measure. Once such a measure is found, one is able to obtain essential information regarding the system, such as the frequency the orbits enter a specific region, the entropy, the Lyapunov exponents, mixing rates etc. I will illustrate this phenomenon with the parametrized family of symmetric doubling maps. This is joint work with C. Kalle.

Sohail Farhangi: *Van der Corput's Difference Theorem and the left regular representation*

Abstract: Van der Corput's Difference Theorem (vdCDT) is a useful tool in the study of multiple ergodic averages. We will show how the classical vdCDT produces sequences that have Lebesgue spectrum in a suitably interpreted sense. We then show that an analogous vdCDT for countable amenable groups produces sequences

that produce subrepresentations of the left regular representation. As applications we will obtain results about multiple ergodic averages for actions of countable abelian groups with noncommuting transformations.

Diego González-Sánchez: *On measure-preserving $\mathbb{F}_p^\omega$-systems of order k*

Abstract: Building on previous work in the nilspace-theoretic approach to the study of Host–Kra factors of measure-preserving systems, we prove that every ergodic $\mathbb{F}_p^\omega$-system X of order k is a factor of an Abramov $\mathbb{F}_p^\omega$-system Y of order k, i.e., $L^2(Y)$ is spanned by the phase polynomials of degree at most k. This answers a question of Jamneshan, Shalom and Tao.

Yonatan Gutman: *The finite-dimensional embedding problem in topological dynamics for arbitrary group actions*

Abstract: According to the classical Menger–Nöbeling (1932) theorem, a compact metric space X of (Lebesgue covering) dimension less than $r/2$ admits a topological embedding into r-dimensional Euclidean space. Generalizing this to the dynamical setting we prove that whenever an (arbitrary) group G acts on a finite-dimensional compact metric space X, there exists an equivariant topological embedding of X into $([0,1]^r)^G$, provided that for every positive integer N, the dimension of the space of points in X with orbit size at most N is strictly less than $Nr/2$. During the talk we will discuss the proof as well as previous proofs given by Gutman (2015) for $G = \mathbb{Z}$ and Gutman, Qiao and Szabó (2018) for G a finitely generated group. We also state, a general embedding conjecture in the generality of sofic group actions. Based on a joint work with Michael Levin and Tom Meyerovitch.

Joanna Kułaga-Przymus: *One measure to rule them all: convolution systems*

Abstract: Convolution systems are a family of 0-1 subshifts defined in terms of invariant measures. Namely, let M be the coordinatewise multiplication of two 0-1 sequences. A subshift $X \subseteq \{0,1\}^{\mathbf{Z}}$ is called a convolution system if there exists a measure ν such that the set of Borel probability invariant measures on X consists precisely of measures of the form $M_*(\rho)$, where ρ lives on $\{0,1\}^{\mathbf{Z}} \times \{0,1\}^{\mathbf{Z}}$ and projects onto the first coordinate to ν. The motivation to study such systems comes from the theory of $\mathscr{B}$-free subshifts. Let $\mathscr{B} \subset \mathbb{N}$ and let $\eta = \mathbf{1}_{\mathbf{Z} \setminus \bigcup_{b \in \mathscr{B}} b\mathbf{Z}}$. Let X_η be the orbit closure of η under the left shift. It is known that $\widetilde{X}_\eta := M(X_\eta \times \{0,1\}^{\mathbf{Z}})$ is then a convolution system. I will also talk about bi-convolution systems which are a certain generalization of convolution systems. Examples include X_η under some natural assumptions on $\mathscr{B}$. Based on joint work with Michał Lemańczyk and Michał Rams.

Daniel Lenz: *Aperiodic order and mathematical diffraction theory*

Abstract: The discovery of quasicrystals by diffraction experiments by Shechtman in 1982 came completely unexpected to people working in physics, chemistry and material sciences alike. The mathematical description lead to the field of aperiodic order and mathematical diffraction theory. Central objects in mathematical diffraction theory are the autocorrelation and its Fourier transform the diffraction

measure. A basic question concerns criteria for the diffraction to be a pure point measure. This and related questions can be studied with the help of unitary representations. One such representation comes from the Koopman operators associated to a certain dynamical system. We give an introduction into this topic.

Michael Lin: *Uniform ergodicity and the one-sided ergodic Hilbert transform*

Abstract: Let T be a bounded linear operator on a Banach space X satisfying $\|T^n\|/n \to 0$. We prove that T is uniformly ergodic if and only if the one-sided ergodic Hilbert transform $H(T)x := \lim_{n\to\infty} \sum_{k=1}^{n} k^{-1}T^k x$ converges for every $x \in (I-T)X$. When T is power-bounded (or more generally (C,α) bounded for some $0 < \alpha < 1$), then T is uniformly ergodic if and only if the domain of H equals $(I-T)X$. Joint work with Guy Cohen.

Pablo Lummerzheim: *Ergodicity of Skew-Products over Markov chains*

Abstract: We study bundle random dynamical systems, where a stationary Markov chain picks randomly a measure preserving transformation acting on some probability space. Using the kernel operator associated to the Markov chain we extend a criterion of Alexander I. Bufetov for the ergodicity of such random dynamical system from Markov chains with finite state space to Markov chains to general state space. We also show that the criterion is in fact a characterization. We shall then use this to prove two random ergodic theorems. This talk is based on a joint work with Felix Pogorzelski and Elias Zimmermann.

Mostafa Sabri: *Ergodic theorems for unitary evolutions on tori and crystal lattices*

Abstract: I will give several quantum dynamical analogs of the classical Kronecker-Weyl theorem, which says that the trajectory of free motion on the torus along almost every direction tends to equidistribute. The quantum analog we propose is to study the Laplacian semigroup evolution $exp(-i\Delta)\psi$ applied to a localized initial state ψ. Then the evolution will be ergodic if this evolved state becomes equidistributed on the torus as time goes on. We prove that this is indeed the case for evolutions on the flat torus, provided we start from regularized Dirac distributions, and we also prove discrete analogs of this result for crystal lattices such as $\mathbb{Z}^d$. On the other hand, we show that this property is violated on the sphere, which is in harmony with the fact that classical free motion on the sphere simply draws a great circle (so, very far from equidistributing). This is based on a joint work with Anne Boutet de Monvel.

Daniel Sell: *Invariant B-free measures from codings*

Abstract: For a set B of natural numbers, the characteristic function of B-free numbers defines a 0-1-subshift, called the B-free subshift. As discussed in the talk of Joanna Kułaga-Przymus, some B-free subshifts are bi-convolution systems. Thus the subshift's invariant measures can be described in terms of so-called Mirsky measures. However, those B-free subshifts can also be thought about via a coding from a certain group. It was conjectured by Keller that the invariant measures on a B-free subshift hence can be described in terms of this group's Haar measure. In the talk, we discuss that the conjectured form can indeed be obtained from the aforementioned result about the Mirsky measures. This is joint work with Aurelia Dymek and Joanna Kułaga-Przymus.

Ivan Veselic: *Quantitative uniform convergence estimates for the integrated density of states.*

Abstract: The integrated density of states is used in quantum solid states physics to measure/count the electron energy levels per unite volume. It can be defined in two ways: By means of a close formula involving mathematical expectation and by an exhaustion procedure relying on some sort of law of large numbers. The underlying convergence has been studies for decades, more recently also in uniform topology w.r.t. the energy parameter.

We present results with explicit and quantitative error terms based on concentration inequalities by Talagrand, Pollard, van den Vaart, Wellner.

This is joint work with Max Kämper, Christoph Schumacher and Fabian Schwarzenberger.